From Electronics to Microelectronics
W.A. Kaiser and W.E. Proebster (eds.)
© North-Holland Publishing Company, 1980

A WIDE RANGE STATIC INVERTER SUITABLE FOR INDUCTION MOTOR DRIVES, DIGITAL MODEL AND TEST RESULTS

M.A. Shimy MANSOUR, M.A. ALHAIDER & I. FATOH
Electrical Engineering Department
College of Engineering
Riyad University
Riyad, Saudi Arabia

This paper discusses, develops and compares the theoretical and experimental work for a three phase inverter induction motor system. The new and practical digital model is developed and arranged to solve the inverter induction motor for steady state or transient in case of symmetrical and unsymmetrical triggering with more accuracy than others [1]. On the basis of the new digital model developed in this paper, a method for simulation of controlled-slip variable-speed inverter induction motor drive system is described and it can be used for optimization and predicting the dynamic behaviour of drives and control systems using induction motors. The experimental verification of the digital computer results is presented. A set of phase current, D.C. input current and output torque oscillograms are presented for different synchronous frequencies and load torques. Constant slip frequency characteristic is obtained and compared with the results obtained by theoretical simulation. Also, unsymmetrical triggering conditions are tested for different load torques.

1. INTRODUCTION

The inverter induction motor systems are becoming increasingly practical. Therefore, a need arises for an improved digital model for calculating the dynamic and steady-state behaviour of such systems.

Because there are many constraints imposed on induction motor phases that change many times during one period, considerable difficulty is encountered in analyzing the electrical and mechanical transient or steady state conditions.

All the available methods of multiple reference frames, equivalent circuit approach and others [1] assume the shape of the phase voltage. In realty, then, such methods use the ideal inverter voltage in case of resistive load as an ideal voltage for the inductive load and induction motor load.

The investigation shows that the assumption of a known voltage, as in case of resistive load, is not sufficiently accurate for current waveform and instantaneous torque determination [2].

A new digital model by means of tensor analysis, is proposed to calculate the inverter induction motor system behaviour without the need to assume the shape of the phase-voltage. Hence the model is generalized to handle other cases such as converter-inverter induction motor system and cycloconverter-induction motor system considering filter and source impedances.

The pulsation torque is investigated for different cases and compared in case of symmetrical and unsymmetrical operation.

1.1 The Basic Equation

The following voltage equation is applicable to each winding of stator and rotor of three phase induction motor

$$V = p\lambda + Ri \tag{1}$$

The basic equation is obtained by using the concept of power invariance [3]. Its basic tenet is to transform the time dependent equation (1) to time independent equation in a way similar to the direct and quadrature transformations (1c, 1f). However, the transformation is done without change in stator variables so that (a) these quantities may easily be obtained & (b) the forced inverter terminal constraints could be applied to the equation. The variable stator-to-rotor mutual inductances can be made constant by transforming the rotor quantities to a stationary reference frame.

The equations of five variables are obtained without distorting the physical meaning. These variables are as of the following equation:

$$[V]_p = [G_{(w^r)}]_p\ [i]_p + [L]_p\ p[i]_p \tag{2}$$

This equation is expressed in a form, where the normal constraints upon the voltage of the machine imposed by the known inverter switching connections, are immediately applicable. In addition, the solution for the dependent current variables can be easily interpreted in terms of the phase current. Therefore, equation (2) is taken as a primitive equation for inverter-induction motor system. With the basis of tensor analysis, the phase voltage is not assumed as the in previous methods [1].

1.2 Method of Analysis

With many inverter modes, the voltage applied to the motor phases is obtained by a series of thyristor switchings for the phases across a D.C. source periodically. The most common example is the six interval inverter system. The motor equation for each interval can be obtained by using the connection matrix between the primitive system and the interval system as in the following:

Interval "1"

Thyristors 1, 4 and 6 in fig. (1) are ON as shown in fig. (2b). The rotor has the same connection for all intervals similar to the primitive connection. Therefore, the connection matrix $[c]_1$ between the primitive variables in equation (2) and the new variables shown in fig. (2b), can be written by comparing fig. (2a) with fig. (2b) as:

$$[i]_p = [C]_1\ [i]_1 \qquad (3)$$

or

$$\begin{pmatrix} i_\alpha \\ i_\beta \\ i_a \\ i_b \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_a \\ i_b \end{pmatrix} \qquad (4)$$

Applying the tensor basis [4] to equations (2) and (3) yields:

$$[V]_1 = [G_{(w^r)}]_1\ [i]_1 + [L]_1\ p\ [i]_1 \qquad (5)$$

where

$$[V]_1 = [C]_1^T\ [V]_p = \begin{pmatrix} V_A - V_C \\ V_B - V_C \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} E \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad (5a)$$

$$[G_{(w^r)}]_1 = [C]_1^T\ [G_{(w^r)}]_p\ [C]_1 \qquad (5b)$$

and

$$[L]_1 = [C]_1^T\ [L]_p\ [C]_1 \qquad (5c)$$

Interval "6"

Thyristors 1, 2 and 3 in fig. (1) are ON as shown in fig. (2c). The equation will be:

$$[V]_6 = [G_{(w^r)}]_1\ [i]_6 + [L]_1\ p[i]_6 \qquad (6)$$

where

$$[V]_6 = \begin{pmatrix} V_A - V_C \\ V_B - V_C \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -E \\ 0 \\ 0 \end{pmatrix} \qquad (6a)$$

Intervals 1 and 6 are presented here as samples. Similarly equations for intervals 2, 3, 4 and 5 can be derived.

1.3 Mathematical Model

A general model is proposed to solve the inverter induction motor system permitting a convenient and simple method of analyzing steady state modes considering speed pulsation in case of symmetrical and unsymmetrical triggering as well as transient processes.

Equations (5) through (6) can be written as:

$$p[i]_n = [L]_1^{-1}[G_{(w^r)}][i]_n + [L]_1^{-1}\ [V]_n \qquad (7)$$

The solution of equation (7) for any time $(n\Delta t)$ expressed in terms of the system initial conditions at $(n-1)\ \Delta t$ is [1e]

$$[i_{(n)}]_n = e^{[A_{(n)}]\Delta t}\ [i_{(n-1)}]_{n-1} + [U] - e^{[A_{(n)}]\Delta t}\ [G_{(n)}]^{-1}\ [V]_n \qquad (8)$$

where

$$[A_{(n)}] = -[L]_1^{-1}\ [G_{(n)}]_1 \qquad (8a)$$

and

$$[G_{(n)}] = [G_{w^r_n}]_1 \qquad (8b)$$

To apply equation (8), it is necessary to find the initial current vector $[i_{(n-1)}]$ assuming the speed is constant for each interval. Equation (8), can be used with more accuracy when each interval is devided to subintervals (K) as

$$[i_{(nK+m)}]_{n+1} = e^{[A_{(nK+m)}]\Delta t}\ [i_{(nK+m-1)}] + [U]-e^{[A_{(nK+m)}]\Delta t}\ [G_{(nK+m)}]^{-1}\ [V]_{n+1} \qquad (9)$$

where m = 0, 1, 2, ... K subintervals
and n = 0, 1, 2, ... N intervals

Once the induction motor currents are calculated, the inverter input currents can be evaluated easily by using the interval relation matrices 1 to 6 derived from fig. (2) as follows:

$$i_{in(1)} = [1 \quad 0] \begin{pmatrix} i_1 \\ i_2 \end{pmatrix}_1, \quad i_{in(2)} = [1 \quad -1] \begin{pmatrix} i_1 \\ i_2 \end{pmatrix}_2 \qquad (10a,b)$$

$$i_{in(3)} = [0 \quad -1] \begin{pmatrix} i_1 \\ i_2 \end{pmatrix}_3, \quad i_{in(4)} = [-1 \quad 0] \begin{pmatrix} i_1 \\ i_2 \end{pmatrix}_4 \qquad (10c,d)$$

$$i_{in(5)} = [-1 \quad 1] \begin{pmatrix} i_1 \\ i_2 \end{pmatrix}_5, \quad i_{in(6)} = [0 \quad 1] \begin{pmatrix} i_1 \\ i_2 \end{pmatrix}_6 \qquad (10e,f)$$

where $i_{in(1)}$ to $i_{in(6)}$ represent the currents of interval 1 to 6 respectively.

2. APPLICATION OF THE NEW DIGITAL MODEL

Interesting applications for equations (9) & (10) by using the digital computer are transient and steady-state in case of symmetrically or unsymmetrically triggered thyristors as in the following items:

2.1 Symmerically Triggered Thyristors

For symmetrically triggered thyristors equation (9) is used with equal interval times. In addition to equation (9), the dynamic behaviour of the system can be studied by the equation governing the change of rotor speed as:

$$p\dot{\theta}_n = \left([i_{(n)}]^T[G_{(n)}]-[R][i_{(n)}]\right)_w \frac{P}{rJ} - \frac{T_L}{J} \qquad (11)$$

The differential equation (11) can be arranged in a form suitable for digital computer by using any numerical method such as Runge-Kutta method or Predictor-corrector method, etc. If the Predictor-corrector method is used, equation (11) will be in the form:

$$\dot{\theta}^{o}(nK+m+1) = \dot{\theta}(nK+m) + \Delta t \, p \, \dot{\theta} \, (nK+m) \qquad (12)$$

$$\dot{\theta}^{1}(nK+m+1) = \frac{\dot{\theta}(nK+m)+\dot{\theta}^{o}(nK+m+1)}{2} + \frac{\Delta t}{2} p \dot{\theta}^{o}(nK+m) \qquad (13)$$

$$\dot{\theta}^{k}(nK+m+1) = \frac{\dot{\theta}(nK+m)+\dot{\theta}^{k-1}(nK+m+1)}{2} + \frac{\Delta t}{2} \, p \, \dot{\theta}^{k-1} \, (nK+m) \qquad (14)$$

The iteration is terminated when two successive iterates agree to the desired accuracy.

The system of equations (9), (12), (13) & (14) can be solved by introducing the initial values for the currents and rotor speed. Hence, these equations are suitable to solve any sudden change. Also it can be used to get the steady-state solution whether the speed is constant or not.

2.2 Unsymmetrically Triggered Thyristors

For practical reasons the inverter-induction motor system may have unsymmetrically triggered wave as shown in fig. (3). Equations (9), (12), (13) and (14) must be modified so that the lengthened interval and the shortened interval have the correct number of sub-intervals. This may be done as follows, for example, take interval (n) as lengthened one and (n+1) as a shortened one as shown in fig. (3). The program must be designed so that interval (n) has a number of subintervals higher than K and interval n+1 has a number of sub-interval less than K. By doing this the solution is obtained using equations (9), (12), (13) and (14).

2.3 Application to the Analysis of a Converter-Inverter Induction Motor Drive

The new digital model is employed to establish a method of calculating the inverter-induction motor variables which are supplied by any voltage shape. In converter-inverter system, the input inverter voltage depends on the inverter-induction motor variables because of filter and source impedences as shown in fig. (4). The equivalent circuit and equivalent source voltage are shown in fig. (5). The equation describing the system shown in fig. (5), is derived as follows:

$$\bar{V}_{s(n)} = (R + Lp)(i_{in(n)} + C_f \, p\varepsilon) \qquad (15)$$

where

$$R = R_s + R_f \quad \text{and} \quad L = L_s + L_f \qquad (15a)$$

The inverter input currents can be expressed in terms of the motor phase currents by using equation (10) to be in the following form:

$$i_{in(n)} = [C]_n \, [i]_n \qquad (16)$$

Combining equations (15) and (16) and using equation (7) for the vector P[i] yield:

$$\bar{V}_{s(n)} = R[C]_n + L[C]_n \, [L]^{-1}[G_{(w^r)}] \, [i]_n + L[C]_n [L]^{-1}[C_v]_n \, \varepsilon + RC_f p\varepsilon + LC_f p^2 \varepsilon \qquad (17)$$

$$\text{where,} \quad [C_v]_n \varepsilon = [V]_n \qquad (17a)$$

The second order differential equation (17) can be written as a set of two first-order differential equations by using the state-space approach (5). A set of state variables sufficient to describe this system is the inverter terminal voltage (ε) and the rate of change of it ($p\varepsilon$). Therefore, we will define a set of state variables as (x_1, x_2) where:

$$x_1(t) = \varepsilon(t) \quad \text{and} \quad x_2(t) = p\varepsilon(t) \qquad (18)$$

Equation (17) can be written in terms of the state variables in a matrix form as follows:

$$p \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ -\frac{1}{C_f}[C]_n [L]^{-1}[C_v]_n + \frac{1}{C_f}[C]_n [L]^{-1}[G_{(w^r)}] & -\frac{R}{L} & -\frac{R}{LC_f}[C]_n \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ [i]_n \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{1}{LC_f} \end{pmatrix} \bar{V}_{s(n)} \qquad (19)$$

Equations (19), (7) and (10) form a perfect and complete model for converter- inverter induction motor system, which can be solved as follows:

The forcing function ($\bar{V}_S$) is known according to the known A.C. supply voltage and the converter firing angle (α). Because of converter effect, this forcing function ($\bar{V}_S$) is a sinusoidal function that is repeated every half cycle with respect to the frequency of the A.C. supply. The acting time (t_c) of the forcing function ($\bar{V}_S$) begins with the firing time of the converter and it continues as long as the current through the conducting thyristor is positive. Once the thyristor current goes to zero, the forcing function becomes zero and will continue to be zero until the start of the second firing time and so on. The thyristor conduction time (t_c) is affected by the forcing function which is the load torque.

In short, the system of equations (7), (10) & (19) are summarized as a set of simultaneous differential equations as follows:

$$p \begin{pmatrix} x_1 \\ x_2 \\ i \\ \theta \end{pmatrix} = \begin{pmatrix} f(x_1, x_2, [i], \dot{\theta}, t) \end{pmatrix} + \begin{pmatrix} f(\bar{V}_s, T_L, \alpha) \end{pmatrix} \qquad (27)$$

The recommendations for this model is obtained by using it to solve the inverter-induction motor system considering the D.C. link resistance and inductance. To consider the filter and other series impedence effects, it is necessary to use this newly developed digital model. This model enables us to predict the interaction between the load and the source, especially for converter-inverter system.

3. CONCLUSION

An inverter requires direct knowledge of the state of its phase quantities. The proposed transformation of the machine equations retains the stator phase variables while eliminating the time and position dependence of the machine inductance parameters. Therefore, this transformation expedites the matching of the output terminal conditions of the inverter to the input terminal conditions of the induction motor and this in turn expedites the matching of the output terminal conditions of the converter or the DC source to the input terminal conditions of inverter for any operation type, while retaining computational simplicity. Thus it can be seen that the model is useful for evaluating the transient and dynamic performance of the induction motor in case of symmetrical or unsymmetrical inverter operation. The model lends itself to easy manipulation, taking into account the source constraints and operation conditions of the converter and inverter. Detailed equations derived in forms which are directly soluble on a digital computer and should be useful, along with the interval state technique suggested, in the study of the dynamic and transients in case of symmetrical or unsymmetrical operation of any inverter type and cycloconverter.

This model enables one to consider the D.C. linkfilter easily without approximation. The model is applied for symmetrical and unsymmetrical cases. It shows that the motor torque shape is very senstitive to the unsymmetry of the triggering. The unsymmetry of one interval led to modulation of the six pulses wave by the fundamental pulsation. The unsymmetry for each interval led to a distortion of the sixth pulse wave. These pulsating torques may lead to higher shaft stresses and speed oscillation.

The experimental torque oscillogram in case of imperfect symmetrical triggering, as shown in fig. (6), has the first harmonic pulsation torque superimposed by a sixth harmonic torque. The experimental oscillogram in case of perfect triggering, as in fig. (8) has the sixth harmonic torque as that obtained by the digital computer as shown in fig. (7).

The above results show a good recommendation for the new digital model approach. Hence, it may be used to detect the reasons behind the unexpected torque pulsation as in fig. (6). Figures (9), (10), (11) are computed for different types of unsymmetry. These figures namely (9), (10) and (11) show that the motor torque shape is very senstitive to the unsymmetry of the triggering. In fig. (11), for example, unsymmetry of (2.5°) for one interval led to modulation of the six pulses wave by the fundamental pulsation with amplitude of 200 per cent compared with the six pulses torque amplitude. In fig. (10), however, unsymmetry of (2.5°) for each interval led to a distortion of the six pulses wave.

LIST OF SYMBOLS

a,b,c	Subscripts referring to rotor phases
A,B,C	Subscripts referring to stator phases
$[A]^{-1}$	Inverse of matrix [A]
C_f	Filter capacitance
$[C]^T$	Transpose of connection matrix [C]
$[C_V]_n$	Defined matrix
E	Inverter dc input voltage
f	Frequency, function
$[G_{(w^r)}]$	Speed dependent impedence matrix
i	Instantaneous current
[i]	General current vector
$[i_{(o)}]$	Initial current vector
$[i_{(n)}]$	Current vector at time interval $(n\Delta t)$
i_a, i_b, i_c	Rotor currents referred to stator
i_a^r, i_b^r, i_c^r	Actual rotor currents
$i_\alpha, i_\beta, i_\gamma$	Stator currents
J & K	Angular moment of inertia & Integer
[L]	Speed independent inductance matrix
L_f	Filter inductance
L_s	Source inductance
L^s	Self inductance of one stator phase
L_p^r	Effective rotor self inductance of one phase referred to rotor
m	Subinterval number
n	Interval number
p	Subscript of referring to primitive parameters
P	Number of pole pairs
$\underline{p}$	$\frac{d}{dt}$
r	Superscript referring to rotor parameters
R	Resistance
R_f	Filter resistance
R_s	Source input resistance
s	Superscript referring to stator parameters
t & t_c	Time and Conduction time of thyristor
Δt	Time interval
T	Electromagnetic torque
T_L & [U]	Load torque and Unit matrix
v	Instantaneous phase voltage
[V]	General voltage vector
$\bar{V}_s$	Forcing function
w	Electrical angular frequency
x_1, x_2	State variables
α	Thyristor firing angle
ε	Inverter terminal voltage
$[\psi]$	Instantaneous inductance matrix
λ	ψi
$\dot{\theta}$	Rotor angular speed
θ	Rotor angular displacement

REFERENCES

[1] (a) W. Charlton, Matrix method for the steady state analysis of inverter fed induction motors, Proc. IEE, Vol. 120, No. 3, 363-365, 1973.

(b) W. Charlton, Matrix approach to steady state analysis of inverter fed induction motors, Elect. lett., Vol. 6, No. 14, 415-416, 1970.

(c) Krause, et al, Method of multiple reference frames applied to the analysis of a rectifier inverter induction motor drive, IEEE Trans. Power App. & Systems, Vol. PAS-88, No. 11, 1635-1641, 1969.

(d) T.A. Lip, et al, Harmonic torque and speed pulsations in a rectifier inverter induction motor drive, IEEE Trans. Power App. & Systems, Vol. PAS 8, No. 5, 579-587, 1969.

(e) Krause, et al, Analysis and simplified representations of a rectifier-inverter induction motor drive, IEEE Trans.Power App. & Syst., Vol.88, No. 5, 588-596,69

(f) Krause, Method of multiple reference frames applied to the analysis of symmetrical induction machinery, IEEE Trans. Power App. & Systems, Vol. PAS-87, No. 1, 218-227.
(g) Eugene A. Klingshim et al, Polyphase induction motor performance and losses on nonsinusoidal voltage sources, IEEE Trans. Power App. & Systems., Vol.PAS-87, 624-631, 1969.
(h) G.C. Jain, The effect of voltage waveshape on the performance of a 3 phase induction motor, IEEE Trans. Power App Systems, Vol. PAS-85, 1-6, 1969.
[2] Linos J. Jocorides, Analysis of induction motor drives with a nonsinusoidal supply voltage using Fourier analysis, IEEE Trans. Industrial Applications, Vol.IA-9, No. 6, 741-747, 1973.
[3] M.L. Liou, A novel method of evaluating transient response, Proc. IEEE, Vol. 54, No. 1, 20-23, 1968.
[4] Stuart D.T. Robertson et al, A digital model for 3 phase induction machines, IEEE Trans. Power App & Systems, Vol. 38, No. 11, 1624-1634, 1969.
[5] Linos J. Jacorides, Analysis of a cycloconverter induction motor drive system allowing for stator current discontinuities, IEEE Trans. Industry Applications, Vol.IA-9, No. 2, 206-214, 1973.

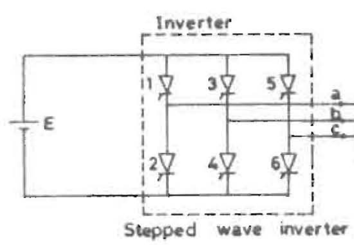


Figure (1)

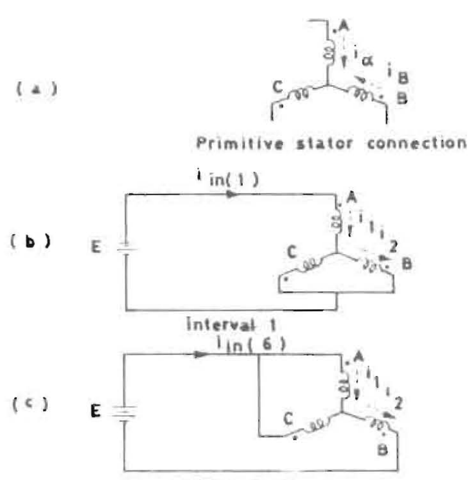


Figure (2) Six interval connections

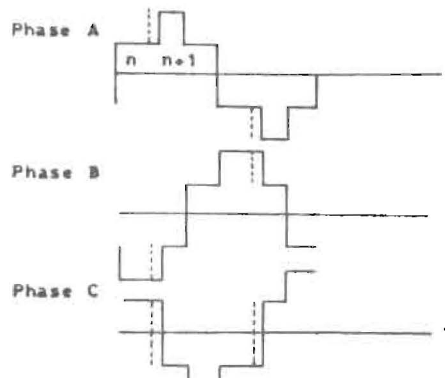


Figure.(3) ---- Symmetrical Triggering
—— Unsymmetrical Triggering

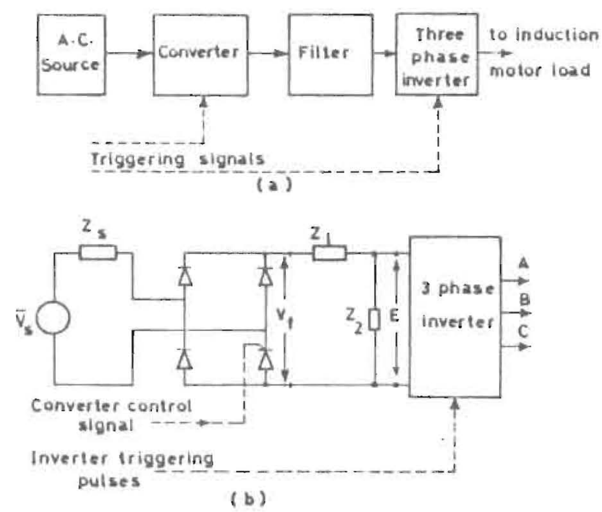
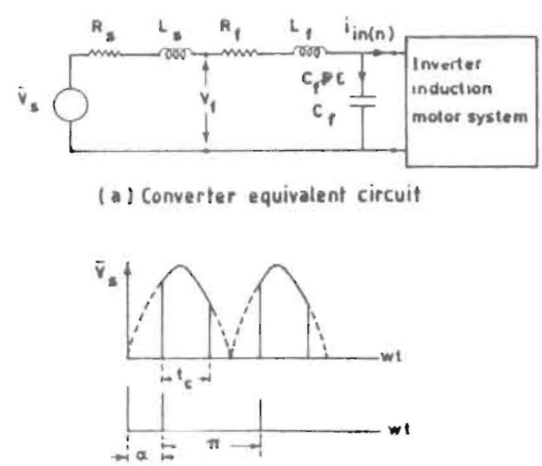


Figure (4) Converter Inverter Induction Motor System



(a) Converter equivalent circuit

(b) Equivalent source voltage wave shape

Figure (5)

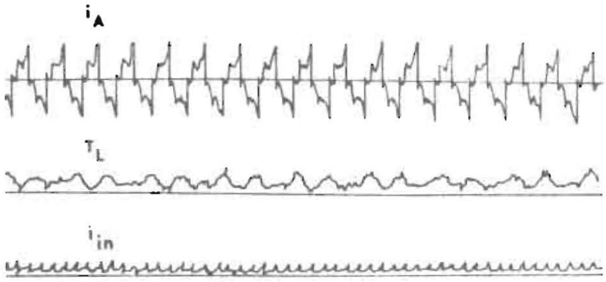


Fig.(6) Experimental Oscillogram for Symmetrical Triggering
F = 50 Hz $\dot{\theta}$ = 1370 rpm

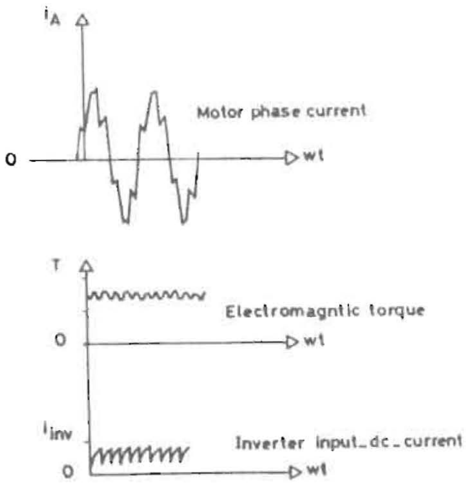


Figure (7) New Digital Model Results for: Symmetrical Triggering
$\dot{\theta}$ = 1370 rpm
F = 50 Hz
E = 180 V

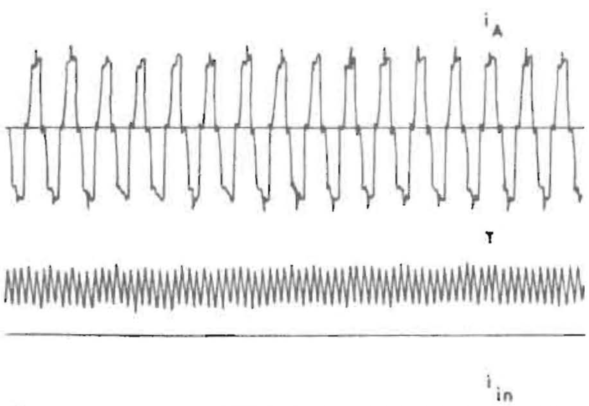


Figure (8) For: Symmetrical Triggering (perfect)
F = 40 Hz $\dot{\theta}$ = 950 rpm

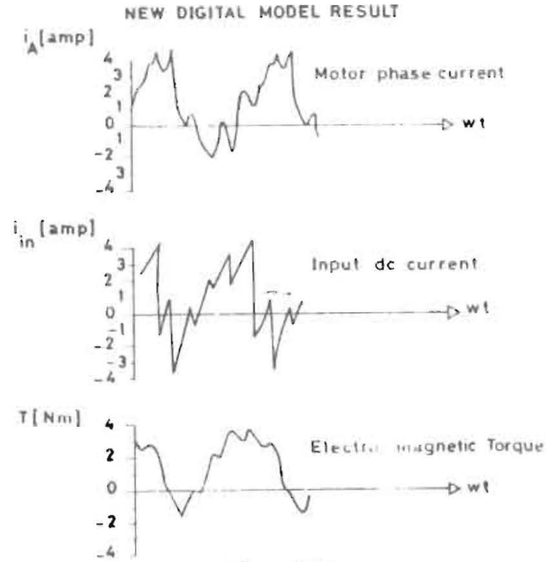


Figure (9)
Unsymmetry by 12° in one step
$\dot{\theta}$ = 1470 rpm F = 50 Hz E = 180 V

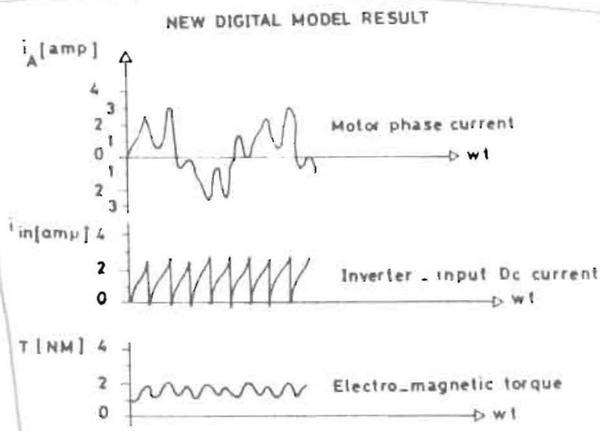


Figure (10) Unsymmetry by 2.5° in each step
$\dot{\theta}$ = 1472.5 rpm F = 50 Hz E = 180 V

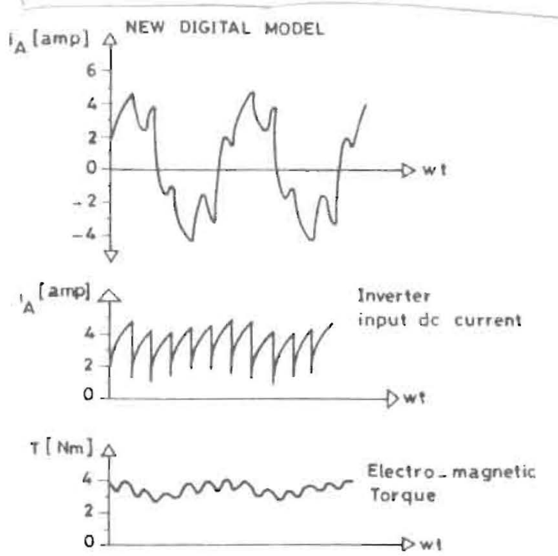


Figure (11) Unsymmetrical by 2.5° per step
$\dot{\theta}$ = 1430 rpm F = 50 Hz E = 180 V

M.A. Shimy Mansour holds the following degrees: B.Sc., M.Sc., Ph.D. (all in Electrical Engineering) and is SMIEEE. He is currently Professor of Electrical Engineering with the College of Engineering, Riyadh University, Riyadh, Saudi Arabia. His areas of interest include Electrical Machines and Power Electronics.

M.A. Alhaider received his B.S. degree from Riyadh University in 1968, the M.S. and Ph.D. degrees from Carnegie-Mellon University, Pittsburgh, PA. in 1972 and 1977 respectively, all in Electrical Engineering. Since 1977, he has been with the College of Engineering, Riyadh University, Riyadh, Saudi Arabia, where he is currently an Assistant Professor of Electrical Engineering and Deputy Dean of the College of Engineering. His areas of interest include Power Electronics, Microwave Communication, Fabrication Techniques and Acoustooptic Interactions in Thin Films and Solar Cells.

I. Fatoh holds the B.Sc. and M.Sc. degrees in Electrical Engineering.